On Structure of Uncertainty
- Categories and Modalities of Uncertainty -

**5 th International Workshop
on Soft Computing Applications**

**22-24 August, 2012
Szeged, Hungary**

Michio Sugeno

**European Centre
for Soft Computing**

Mieres-Asturias, Spain

Content

- **Motivation and Methodology**
- **Some Issues of Language**
- **Evidentness in Cartesian Philosophy**
- **Categories and Modalities of Uncertainty**
- **Three Essential Uncertainties**
- **Structure of Uncertainty**
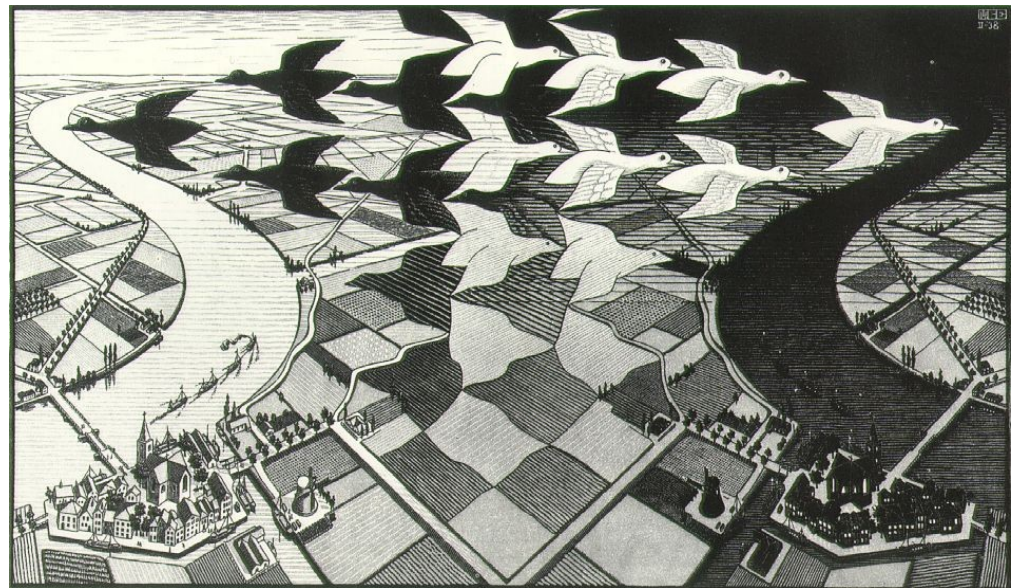- **Conclusions and Future Studies**

Motivation

- **Fuzzy theory has brought a hidden uncertainty, fuzziness, to light which is interpreted as uncertainty concerning the meaning of wording.**

- **Wording has three kinds of uncertainty, viz., ambiguity, fuzziness and nonspecificity (take 'that book'; put it 'there').**

Ambiguity and Fuzziness
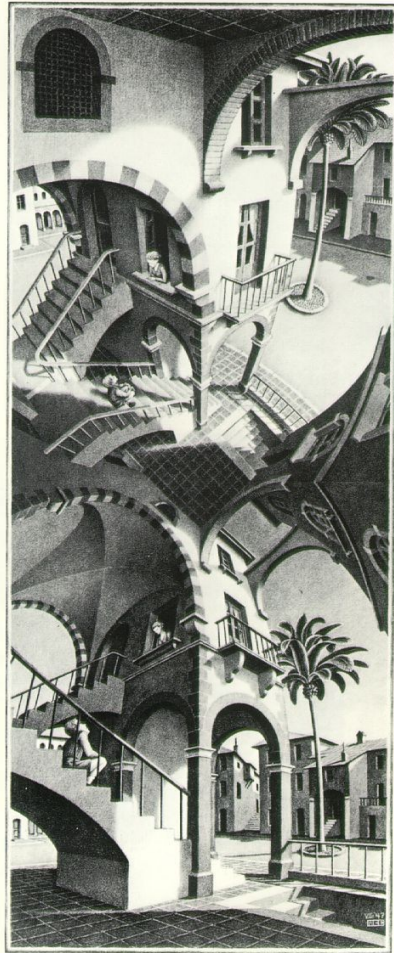
Up and Down (Lithograph 1947)

Escher (1898 - 1972)

Day and Night (Woodcut 1938)

Motivation

- **Fuzzy theory has brought a hidden uncertainty, fuzziness, to light which is interpreted as uncertainty concerning the meaning of wording.**

- **Wording has three kinds of uncertainty, viz., ambiguity, fuzziness and nonspecificity (take 'that book'; put it 'there').**

- **So far, only ambiguity has been considered in natural language processing. Fuzziness is, however, a much more dominant uncertainty than both ambiguity and nonspecificity.**

A fundamental question is:

"What kinds of uncertainty are we aware of ?"

How can we answer this question ?

Methodology of Finding Uncertainties (1)

According to Saussure (1857-1913), the father of modern linguistics, we perceive and think about the world outside us and inside us with language.

We articulate the continuous world into a manageable number of clusters in our perception: discretization of the continuous world with 'language of articulation'.

For instance, a rainbow as a continuous optical spectrum is articulated into a number of colors depending on cultures: two in C. African Republic, three in Zimbabwe, five in Zuni Indian, six in UK & USA or seven in France, Spain & Japan.

Language records in it all the experiences that our species have ever encountered over 100 thousands of years since the brain created modern language.

Development of Human Language
Halliday (1975) & Matthiessen (2004)

Human Species	**2 M years ago** **homo habilis**	**~** **homo gramma-ticus**	**100 K years ago** **homo sapience sapience**
Human Language	**protolanguage bi-stratal:** **content + expression**	**emergence** **of grammar**	**(modern) language tri-stratal: meaning + lexico-grammar +expression**
Human Infants	**10 months ~** **crawling**	**~14 months sitting**	**~ 18 months standing**

Typology of Systems:
Language is the System of the Highest Order

① **Physical Systems**

1×10^9

3.5×10^9

6×10^6

2×10^6

10^5

Years
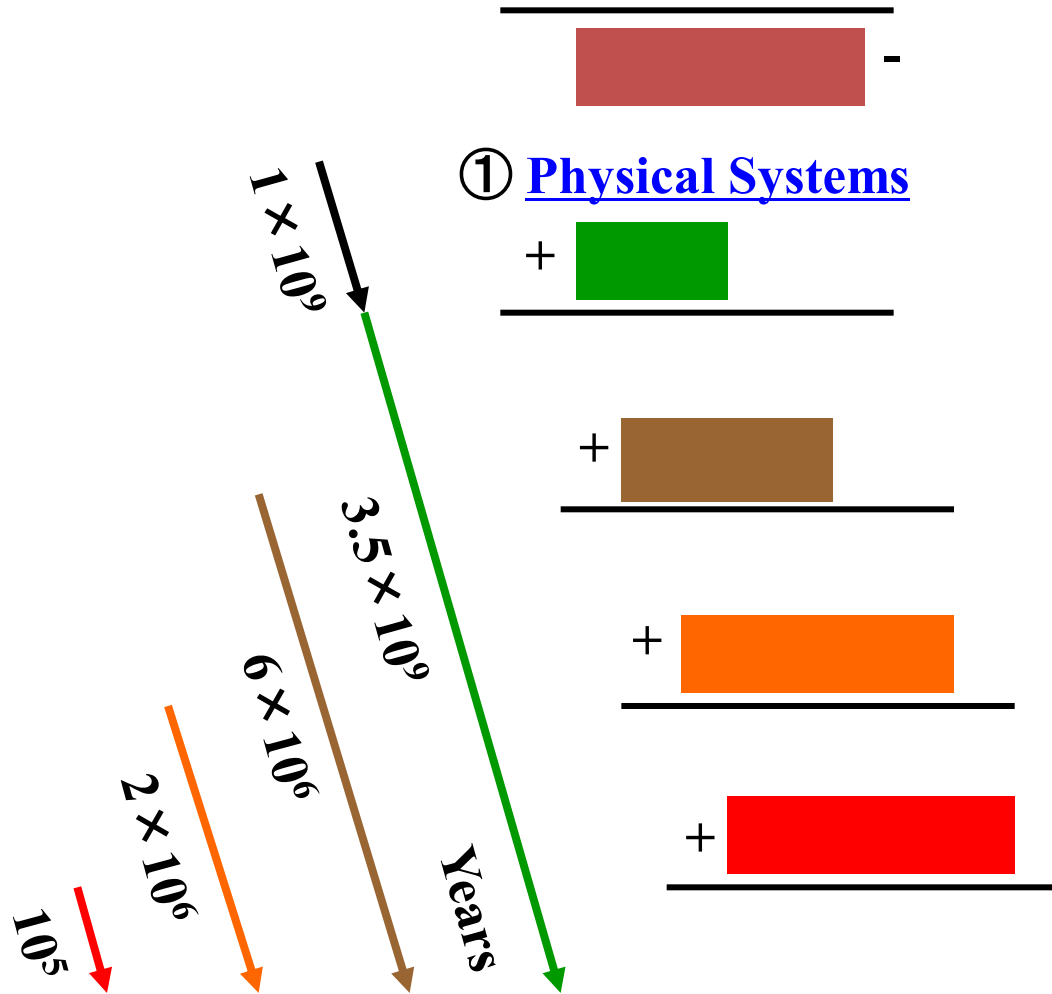
Three Body-Problem in Physics

Two particular solutions are known:
(1) Euler's collinear solution in 1765,
(2) Lagrange's equilateral triangle solution in 1772.

Poincare proved that the problem cannot be analytically solved in 19 C.

'Three' is complex in physics !

K. Yamada and H. Asada gave a collinear solution in the setting of general relativity theory in October, 2010.

Collinear solution to the general relativistic three-body problem

Kei Yamada and Hideki Asada

Faculty of Science and Technology, Hirosaki University, Hirosaki 036-8561, Japan
(Received 31 May 2010; published 8 November 2010)

The three-body problem is reexamined in the framework of general relativity. The Newtonian three-body problem admits *Euler's collinear solution*, where three bodies move around the common center of mass with the same orbital period and always line up. The solution is unstable. Hence, it is unlikely that such a simple configuration would exist owing to general relativistic forces dependent not only on the masses but also on the velocity of each body. However, we show that the collinear solution remains true

Typology of Systems: Language is the System of the Highest Order

⓪ Mathematical Systems

More advanced species ??

− **Energy**

① Physical Systems

1×10^9

+ **Life**

② Biological Systems

3.5×10^9

+ **Value**

③ Social Systems

6×10^6

+ **Meaning**

④ Semiotic Systems

2×10^6

+ **Grammar**

10^5

Years

⑤ Language Systems
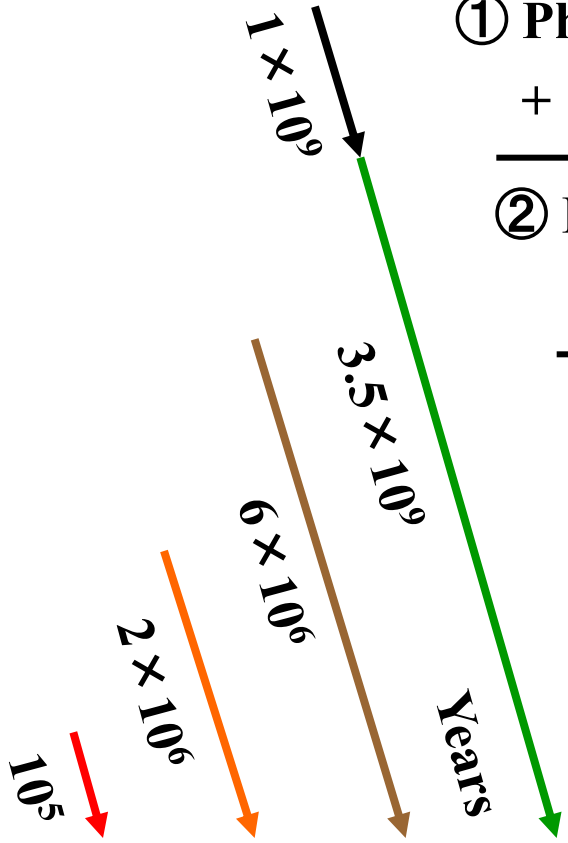
Two Mistakes in Approaching Language

1. **A historical mistake:**
   Russel and Whitehead dreamed to creat an ideal language since ordinary language looked so impresice or vague ('loose concept' in the sense of Frege) to them.

   Their dream was, however, broken when Wittgenstein stated:

   "Odinary language is perfect as it is."

   Known as 'the Linguistic Turn'.

2. **A recent mistake:**
   AI people take a logical and symbolic approch to Language.

   Their efforts will be not fruitful.

Methodology for Finding Uncertainties (2)

- **Found out all the adjectives implying uncertainty by carefully reading Dictionaries of English and Japanese from the first page to the last (in 1986).**

  There were found 170 in English and 150 in Japanese.

- **Then, found out Chinese adjectives by using Japanese-Chinese Dictionary; French and German adjectives as well.**

  There were found 160 in Chinese, 150 in French & German.

- **Last year (in 2011), found out Spanish adjectives referring to both English-Spanish and Japanese-Spanish Dictionaries, and Basque as well. Also recounted English and French adjectives.**

Japanese Adjectives (1)

曖昧な	大方の	かすんだ	主観的
怪しい	おおざっぱな	仮想の	心情的
あやふやな	憶測的	可能な	随意の
ありそうな	憶断的	仮構の	折衷的
いい加減な	起こりそうな	気がかりな	選択的
意外な	おざなりの	近似の	想像の
いかがわしい	覚束ない	偶然の	ぞんざいな
偽りの	おぼろげな	くすんだ	多義的
いぶかりの	おぼろな	傾向の	妥協的
色々な	思いがけない	心もとない	多値的
うさんくさい	およその	混沌とした	偶々な
疑わしい	懐疑的	雑然とした	玉虫色の
うっとうしい	蓋然的	雑多な	多面的
うつろな	確率的	様々の	多様な
うやむやな	架空の	自由な	中間的

Japanese Adjectives (2)

中途半端な	非実在的	不完全の	不整合な
直観の	非組織的	不規則の	不整な
ときたまの	非系統的	不規律な	不鮮明な
突然の	非必然的	複合の	不測の
鈍感な	非明示な	複雑な	不揃いの
任意の	非明証的	不決断の	不確かな
灰色の	非理性的	不見識な	不調和な
漠然とした	非論理的	不思議な	不定の
半信半疑の	不安な	不十分な	不適格な
非科学的	不意の	不詳の	不的確な
非形式的	不可解な	不条理な	不適確な
非系統的	不確実の	不審な	不適当な
非決定的	不確定の	不正確な	不徹底な
非合理的	不可測の	不精確な	不同の

Japanese Adjectives (3)

不透明な   紛らわしい   予想外の
不特定の   まばらな   両義的
不備な   未確認の
不分明の   未完の   **Total 152**
不明確な   未決の   (Sugeno 1986)
不明な   未詳の
不明白な   未定義の
不明瞭な   無定見な
不慮の   無辺の
放心状態の   無目的の
茫然とした   申訳の
ぼけた   朦朧たる
没論理的   模糊たる
ほのかな   ゆううつな
ぼやけた   優柔不断な

English Adjectives (1)

abrupt	anonymous	blurry	complex
absent	antsy	boundless	complicate
absurd	anxious	bottomless	complicated
accidental	approximate	bogus	confounding
adaptable	approximative	bumpy	conflicting
adjustable	arbitrary	byzantine	confused
aimless	arcane	causal	confusing
aleatoric	atypical	capricious	conjectural
aleatory	beclouded	chance	contingent
almost	bizare	changeable	convoluted
alterable	blankly	changeful	cottony
ambiguous	bleary	chaotic	cranky
amisamorphous	blind	characterless	criptic
anarchic	blunted	cluttered	dark
anomalous	blurred	cloudy	darkish

English Adjectives (2)

darkling	disagreeing	doubting	expressionless
deceptive	disarranged	doubtful	evasive
deceitful	disarrayed	dreamy	fabulous
deceiving	disbelieving	dubious	faceless
defective	discrepant	dubitable	faint
deficient	disharmonious	dull	fake
deluding	dishonest	dulled	fallacious
delusive	disordered	eclectic	fallible
delusory	disorderly	elastic	false
desultory	disputable	elliptical	fanciful
deviate	distrustful	endless	fantasied
devious	dithery	enigmatic	fantastic
different	diverse	equivocal	fathomless
dim	double-edged	erroneous	featureless
disagreeable	doubtable	erratic	fickle

English Adjectives (3)

fictional	gloomy	immeasurable	incommunicable
fictitious	half-way	impalpable	incompatible
fishy	haphazard	impenetrable	incomplete
flicker	hazy	imperfect	incomprehensible
flighty	horizonless	implausible	incomprehensive
flucky	hypothetical	improper	inconceivable
fluctuating	illegible	inaccurate	inconclusive
fluid	illegitimate	inadvertent	Incongenial
foggy	Illimitable	inapt	incongruent
fortuitous	illogical	inapposite	incongruous
fragmental	imaginal	inapprehensive	inconsecutive
fragmentary	imaginary	inappropriate	inconsequent
fugitive	imaginative	inarticulate	inconsequential
fuzzy	imagined	incidental	inconsistent
giddy	imitative	incognizable	inconsonant

English Adjectives (4)

inconspicuous	indistinguishable	insecure	jumbled
inconstant	ineligible	insensible	labyrinthian
incoordinate	inequable	insufficient	lax
incorrect	inexact	intricate	lightless
incredulous	inexhaustive	intuitive	likely
indecisive	inexplainable	invalid	limitless
indecipherable	inexplicable	involute	loose
indefinable	inexplicit	involved	loosened
indefinite	inexpressible	irrational	lying
indescribable	inextricable	irrecognizable	mendacious
indeterminate	infelicitous	irregular	messed
indirect	infrequent	irresolute	measureless
indiscernible	inharmonious	kaleidoscopic	miscalculate
indiscriminate	inosculate	knotty	miscellaneous
indistinct	inscrutable	jittery	mimic

English Adjectives (5)

misleading	muzzy	odd	pliable
mistaken	mysterious	outre	possible
mistrustful	nameless	opaque	precarious
misty	nebulous	optional	prepostrerous
mixed	noncommittal	pale	pretend
motley	nondescript	partial	privy
mutable	nonidentical	passable	problematic
mythical	nonrational	peculiar	probable
modifiable	nonsensical	perfunctory	promiscuous
muddled	nonspecific	perturbed	quaint
multiple	obscure	phantasmal	quaking
multivocal	obscured	phantasmic	queer
murky	obtuse	phantom	queerish
mussy	occasional	piebald	questionable
mutable	occult	plausible	questioning

English Adjectives (6)

quirky	shapeless	subjective	unbelievable
reeling	sightless	sudden	unbelieving
relative	slack	sundry	unbounded
repugnant	slackened	suppositional	uncanny
roiled	snippy	suppositious	uncertain
rough	stolid	suspect	unclear
scarce	shaky	suspecting	uncongenial
scattered	smeary	suspicious	unconvinced
selective	smudgy	tangled	unconvincing
senseless	soft	tenebrous	undecided
shaded	specious	tottering	undecipherable
shadowed	spooky	trustless	undefined
shadowy	sporadic	turbid	undetermined
shady	spurious	unaccountable	undiscerning
shaking	strange	unbecoming	undistinguishable

English Adjectives (7)

uneasy	unjust	unreliable	untrustworthy
uneven	unjustifiable	unresolved	untypical
unexpected	unknown	unsettled	upset
unexplainable	unlikely	unshaped	vague
unfaithful	unlimited	unshown	variable
unfathomable	unnamed	unspecified	varicolored
unfathomed	unnotable	unstable	varied
unfit	unnoticeable	unstructured	variegated
unfixed	unplanned	unsuitable	various
unforeseen	unpredictable	unsure	visionless
unidentified	unquiet	unsystematic	voiceless
unimaginable	unreadable	unthinkable	weird
unintended	unreal	untidy	wobbling
unintentional	unreasonable	untrue	wobby
unintelligible	unreasoning	untruthful	wooly

English Adjectives (8)

woozy

worried

stray

undependable

vacillating

versatile

Supplement

bewildering

circuitous

dicey

esoteric

fuliginous

indecipheable

obfuscatory

obnubilated

perplexing

slapdash

Total 436

(Sugeno 1986, 2011)

Basque (1)

absurdu	aukeratze	deskribaezina	ezegonkor
abusurdoa	aurreikusigabea	determinaezina	ezegonkorra
ahul	ausart	edozein	ez-errealista
akastuna	axolagabea	eklektikoa	ezezaguna
aleatorioa	azalekoa	engainagarria	ez-formal
aldakorra	bat-bateko	erabakigabea	ezinezko
ameslaria	behin-behireko	erdibideko	ez-legezko
anaitugabeko	beldurti	erdibidea	ezleiala
anbiguo	bidegabea	erraza	ez-lerrokatua
asatugabeko	bigunak	entzunezin	ez-legitimoa
askea	birtuala	estokastiko	ezohiko
aribitrarioa	borondatezko	eszeptiko	ez-sistematikoa
arraroa	dardarozia	explikaezina	ez-zehatzak
artikulatugabea	desegoki	ez-bistako	ezezaguna
aukerako	definiezina	ezegokia	ez-zientziazkoa

Basque (2)

fidagabea	ilun	kaotikoa	mmalkarra
fikziozko	ilundu	kaskarrak	altzurra
gehienean	iluna	kausazko	misteriotsua
geldo	iluntzeko	kezkagarria	mitxel
gertagarri	imaginarioa	koipetsua	motel
gezurra	inkohenrentziak	konfidantza	mugagabea
gezurrezko	inperfektua	konplexua	nahasgarria
hainbat	inplizitua	kontigenteak	nahasi
helburogabea	intuitiboa	kospiratzaile	nahastu
hutsal	itsu	korapilatsu	nahaspilatua
harrigarria	itzaltsua	lainotsu	nahigabeko
identifikatua	irrazional	lauso	neurrigabe
iheskor	irregularra	lausoa	ohizko
ikusezinezkoa	itzaltsua	lausotu	opakua
illegular	kamuts	makal	ostarteak

Basque (3)

pasable

pattal

polifazetikoa

posible

problematikoa

rala

saken

sinesgaitz inesgarri

subjektiboa

susmagarri

susmagarriak

tartekatua

txarreko

ulergaitz

ulertezina

ustekabeko

zakarra

zalantzati

zalantzazko

zeharkako

zehatz

zehaztugabeak

zehaztugabeko

zentzugabea

zentzugabekeria

zentzurik

zerbait

zihurgabea

zirarrgarria

ziurtasunagabea

zorabiatua

Total 151
(Sugeno 2011)

Spanish Adjectives (1)

abrupto	brusco	desconfiado	dubitativo
absurdo	cambiante	desconocido	dudoso
accidental	causal	deshonesto	emocinal
agitado	caotico	desigual	empañado
alboratado	chapucero	desleal	engañoso
ambiguo	ciego	deordentaso	enigmatico
anonimo	complicado	despistado	equivoco
anioso	confuso	diferente	erroneo
ansioso	conjectural	difuso	eventual
apagado	criptico	discutible	evasivo
aparente	cuestionable	disonante	extraño
aproximado	debil	disparejo	facultativo
atontado	defectuoso	distinto	falible
borroso	delicado	distraido	falso
brumoso	desapercibido	diveso	fantasioso

Spanish Adjectives (2)

ficticio	inpreceptible	incoherente	incorrecto
figurad	imperfecto	incierto	increible
flojo	impreciso	inclassicable	indeciso
fortuito	imprevisible	incomodo	identifinible
gris	inprevisor	incompetente	indefinido
grogui	imprevisto	incompleto	indeterminado
heterogeneo	improbable	incomprensible	indescriptible
hipotetico	improviso	incomprendido	indirecto
ilegible	inacabado	incomunicable	indiscriminado
ilegitimo	inadecuado	inconcluso	indistinguible
ilimitado	inadvertido	inconcluyente	indistinto
ilogico	inagotable	incongruente	inefable
imaginario	inarmonico	inconsciente	inenarrable
imaginativo	inarticulado	inconsecuente	inescurtable
imprenetrable	incalculable	inconstante	inestable

Spanish Adjectives (3)

ininelegible	injustificable	irrazonable	opaco
inepto	injusto	irregular	opatativo
inesperado	innominado	irreconciliable	opcional
inescrutable	inopinad	irresoluto	palido
inexacto	inoportuno	lento	parcial
inexplicable	inquieto	leve	pendiente
inexplicado	intermdedio	libre	personal
inexplorado	intranquilo	misterioso	perdito
expresivo	invalido	magico	posible
inexpresable	invisible	melancolico	precario
inexistente	inseguro	neblinoso	preocupado
indebido	insuficiente	nebuloso	probale
infinito	intrincado	negligente	problematico
informal	intuitivo	oscuro	propenso
infrecuente	irracional	ocasional	polisemico

Spanish Adjectives (4)

raro
recondito
repentino
semiinconsciente
sospredente
sosprechoso
somero
subito
superficial
selectivo
subjectivo
tambaleante
tendencies'
tenue
tubio

variableago
variado
verosimil
varios

Total 199
(Sugeno 2011)

French Adjectives (1)

acceptable
accidentelle
aléatoire
ambigu
ambivalent
anxieux
approximatif
arbitraire
bizarre
brumeux
cahoteux
chancelant
changeant
chaotique
chevrotant

complexe
compliqué
conditionné
confus
conjectural
contestable
contingent
cotonneux
défaillant
défectueux
déficient
déloyal
dépressif
désordonné
désorganisé

discutable
différent
difficile
disharmonieux
distrait
douteux
ébauché
embrumé
émotionnel
équivoque
erroné
étrange
évasif
facultatif
fautif

faux
feint
fictif
flou
fortuit
gris
hésitant
hypothétique
ignorant
illimité
illisible
illogique
illusoire
imaginaire

French Adjectives (2)

imaginatif	incompréhensible	indéterminé	inharmonieux
imparfait	incohérent	indifferent	inintelligible
impatient	inconcevable	indifférentiable	injuste
imperceptible	incongru	indirect	injustifiable
imprécis	inconnu	indiscernable	injustifié
impropre	inconséquent	indistinct	innombrable
impromptu	inconscient	inégal	inopportun
improbable	inconsistant	inexact	inouï
inadéquat	inconstant	inexplicable	inquiet
inapproprié	incorrect	inexplicite	irrationnel
inattendu	incroyable	inexprimable	irréaliste
incertain	indécis	inextricable	irréfléchi
incommensurable	indéfini	infidèle	irrégulier
incompatible	indéfinissable	infini	irrésolu
incomplet	indescriptible	informel	insensible

French Adjectives (3)

insondable	multivoque	quelconque	superficiel
instable	mystérieux	rare	surprenant
insuffisant	nébuleux	rêveur	suspect
intermédiaire	négligent	romanesque	tendancieux
intermittent	nuageux	rugueux	terne
intuitif	obscur	sceptique	tremblotant
intuitionnel	occasionnel	singulier	trompeur
invraisemblable	opaque	sommaire	trouble
lobotomisé	optionnel	songeur	truqué
louche	partiel	soucieux	vacillant
malhonnête	perturbé	soupçonnable	vague
méconnaissable	plausible	soupçonneux	vaporeux
méfiant	précaire	souple	variable
mi-chemin	probable	stochastique	verbeux
multiple	problématique	subjectif	virtuel

French Adjectives (4)

voilé
vraisemblable

Total 182
(Sugeno 1986,
2011)

Three Dominant Adjectives

English	uncertain, obscure, vague	fuzzy
Japanese	不確か(futashika-na), 曖昧な(aimai-na), 漠然とした(bakuzento-shita)	ファジィ (fuzzy)
Chinese	不确定, 模糊, 含糊	模糊
French	incertain, obscure, vague	flou
German	unsicher, unklar, dunkel	fuzzy
Spanish	dudoso, oscuro, vago	borroso
Basque	ilun, itsurazko, zalantazko	lausoa

Dominant adjectives in a language can be found by translating them into another language belonging to a different language family. For instance, oscuro in Spanish is translated into English: dark, obscure, deep, dim, shadowy, dusky, gloomy, murky, dusk, dingy, darkling, darksome, twilight, abstruse, insignificant, vat, pitchy, eery, dissolute, cimmerian, murk, mirk : 22 English adjectives.

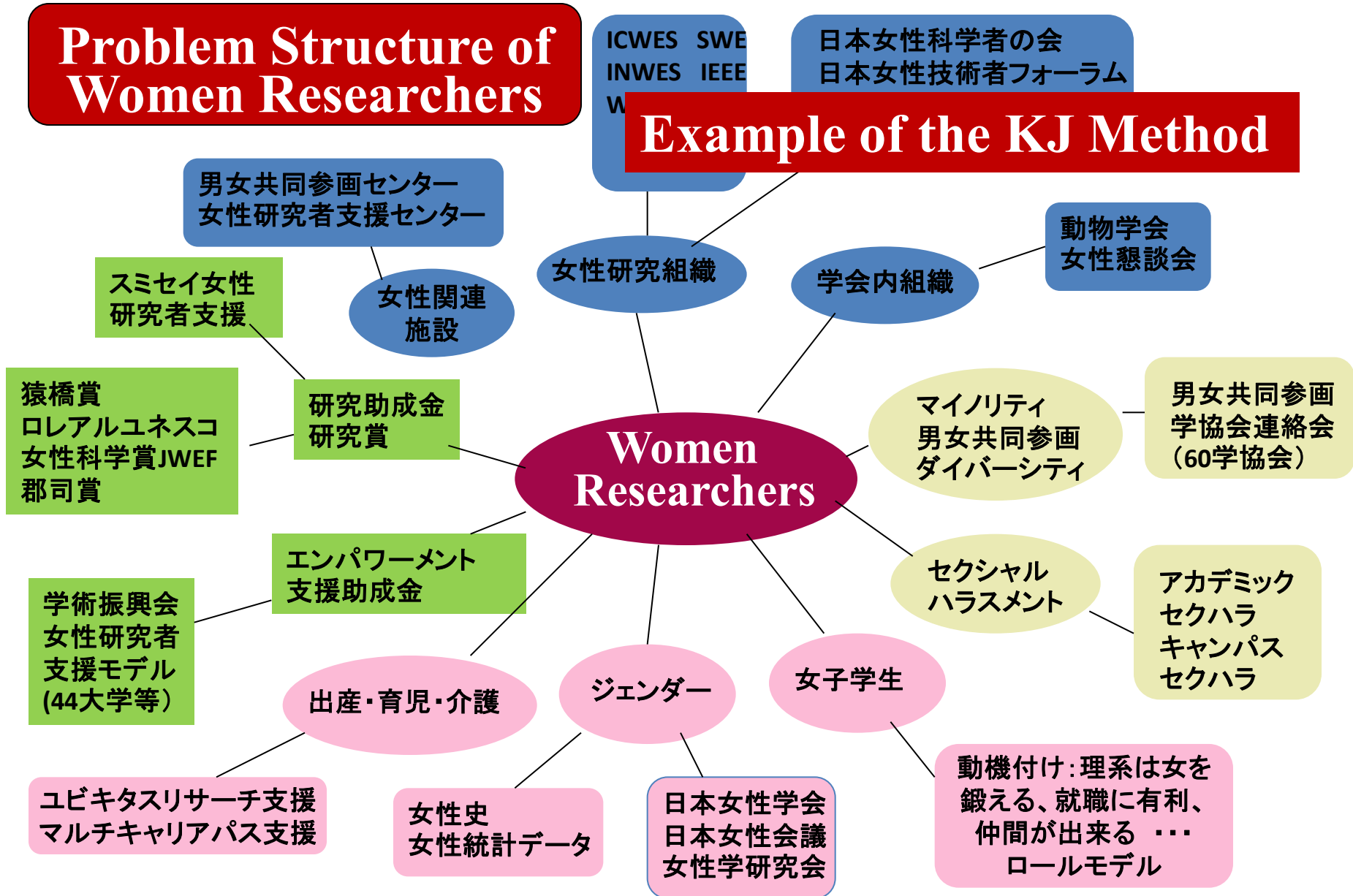# Categories and Modalities of Uncertainty

We use 'uncertainty' and 'uncertain'
as the most general terms in this study.

Classfied the collected adjectives by <u>the KJ method</u> into groups and assign a name to a core-entity which adjectives of a group are intended to →   **category of uncertaity.**
Assigned a modality to a group of adjectives. → **modality.**

Problem Structure of Women Researchers

Example of the KJ Method

ICWES SWE
INWES IEEE
W

日本女性科学者の会
日本女性技術者フォーラム

男女共同参画センター
女性研究者支援センター

女性関連施設

女性研究組織

学会内組織

動物学会
女性懇談会

スミセイ女性
研究者支援

猿橋賞
ロレアルユネスコ
女性科学賞JWEF
郡司賞

研究助成金
研究賞

Women Researchers

マイノリティ
男女共同参画
ダイバーシティ

男女共同参画
学協会連絡会
（60学協会）

学術振興会
女性研究者
支援モデル
(44大学等）

エンパワーメント
支援助成金

セクシャル
ハラスメント

アカデミック
セクハラ
キャンパス
セクハラ

出産・育児・介護

ジェンダー

女子学生

ユビキタスリサーチ支援
マルチキャリアパス支援

女性史
女性統計データ

日本女性学会
日本女性会議
女性学研究会

動機付け：理系は女を
鍛える、就職に有利、
仲間が出来る ・・・
ロールモデル

Categories and Modalities of Uncertainty

We use 'uncertainty' and 'uncertain'
as the most general terms.

Classfied the collected adjectives by the **KJ method** into groups and assign a name to a core-entity which adjectives of a group are intended to → **category of uncertaity.**
Assigned a modality to a group of adjectives. → **modality.**

For instance, **'Existence'** is a category of uncertainty and the modality of it is named **'Non-evidentness'** with reference to Cartesian philosophy.

Adjectives concerning **Non-evidentness of Existence** are:
blurred, chotic, disorderly, disputable, doubtful, fictious, imaginary, impenetrable, inapprehensible, inconspicuous, indescriable, inexpressible, indistinct, limitless, obscure, unclear, etc.

Evidentness (1)

In Cartesian philosophy, 'evidentness' implies both clearness and distinctness. If a concept is given an extensional definition, it is called 'clear', and if it is given an intentional definition, it is called 'distinct'.

Extensional definition: E = { 0, 2, 4, ….. }; clear
Intentional definition: E = { n | n is even }; distinct

'Evidentness' is the first principle of Methodology of Science postulated by Descartes :

(1) evidentness,   (2) analysis,
(3) synthesis,   (4) enumeration.

The first principle claims that we should only deal with evident facts in science.

Evidentness (2)

In his "Philosophical Investigation", Wittgenstein precisely states that the meaning of a word is essentially not evident: neither clear nor distinct.

"How is the concept of a game bounded? ---- The concept 'game' is a concept with blurred edge."

→ imples that the concept of a game is **not clear.**

"Family resemblance of a word: Consider for examples the proceedings that we call games. ---- You will not see something that is common at all. But similarities, relationship, ----."

→ implies that the meaning of a word is **not distinct.**

The meaning of a word is contradictory to the first principle of Cartesian methodology; it is not an object science. **A logical approach will be never successful.**

Concept of Small Number

Define S as a set of small numbers in {1, 2, ..., 10}:

(i) S = {1, 2, 3, ??}; S cannot be extensionally defined.
$\rightarrow$ **not clear.**

(ii) S = { n | n is small}; It is a tautology. S cannot be intentionally defined. $\rightarrow$ **not distinct.**

The concept of small number is **not evident** in the sense of Cartesian philosophy. Therefore, a small number cannot be an object of science; It is right when conventional scientists say "fuzzy theory is not science."

The problem was, however, solved by fuzzy sets.

In a fuzzy set: Small = 1/1 + 0.9/2 + 0.7/3 + 0.4/4 + 0.1/5;
quantitative intention / extension of {1, 2,…, 10}

Paradox of Small Number

Let N = {1, 2, ……, 100} be a universe of discourse.
Consider the following assertions in binary valued logic:

(i)   1 is a small number.
(ii) A small number added by 1 is also a small number.

Then, we conclude that any number, e.g., 100 is small.

This is not acceptable.

Therefore, there is something wrong about the assertions. Since (i) is correct, (ii) must be wrong. That is, there may exist a certain number n^* such that n is small if $n \leq n^*$, and otherwise, not small.

Neither this is acceptable.

Hence, the assertions cause a paradox.

Paradox of Small Number

Let N = {1, 2, ……, 100} be a universe of discourse.
Consider the following assertions in binary valued logic:

(i)   1 is a small number.
(ii) A small number added by 1 is also a small number.
Then, we conclude that any number, e.g., 100 is small.

This is not acceptable.

Therefore, there is something wrong about the assertions. Since (i) is correct, (ii) must be wrong. That is, there may exist a certain number n^* such that n is small if $n \leq n^*$, and otherwise, not small.

Neither this is acceptable.

Hence, the assertions cause a paradox.

Small Number in Fuzzy Logic

Define P_n: " n is a Small Number" and S: a fuzzy set "Small Number" associated with a membership function S(n).
The assertions are now rewritten: (i) P_1 and (ii) $P_n \longrightarrow P_{n+1}$.

Let $|P_n| = S(n)$ conventionally, then apparently $|P_1| = 1$.
Since (ii) is not completely true, set $|P_n \longrightarrow P_{n+1}| = 1 - 0.01$.
Let $|P_n \longrightarrow P_{n+1}| = (1 - |P_n| + |P_{n+1}|) \wedge 1$ in Lukasiewicz logic.
Then, it follows that $|P_2| = 1 - 0.01$ and $|P_3| = 1 - 0.02$.

Solving the equation recursively, we obtain $|P_n| = 1 - 0.01n$.
Therefore, the membership function is now found to be

$S(n) = 1 - 0.01n$, where $S(0) = 1$ and $S(100) = 0$.

Two assertions are a definition of Small Number!

Categories, Modalities and Adjectives

Categories	Modalities	Adjectives
C1A: Existence C1B: Phenomenon	M1A: Non-evidentness M1B: Probability	various. chaotic aleatoric, probable
C2: Consciousness	M2: Vagueness	confusing, dreamy
C3: Wording	M3: Fuzziness	unclear, indefinable
C4: Cognition	M4: Indistinctness	dubious, indiscernible
C5: Action/Behaviour	M5: Nondeterminicity	indecisive, indeterminate
C6: Knowledge/ Information	M6: Uncertainty	unknown, unreliable
C7A: Logic C7B: Form	M7A: Illogicalness M7B: Inconsistency	illogical, inconsequent incomplete, irregular

Alternatives of Categories

Categories	Other choices
C1A: Existence C1B: Phenomenon	None None
C2: Consciousness	Idea, Feeling, Mind
C3: Wording	Word/Concept
C4: Cognition	None
C5: Action/Behaviour	Will, Intention
C6: Knowledge/Information	C6A: Knowledge, C6B: Information
C7A: Logic C7B: Form	None None

On Modalities

Modalities	Note
Non-evidentness of Existence Probability of Phenomenon	**From Cartesian philosophy. No other choice.**
Vagueness of Consciousness	**No other choice.**
Fuzziness of Wording	**Unclearness is a choice.**
Non-distinctness of Cognition	**From Cartesian philosophy.**
Nondeterminicity of Action/ Behaviour	**Indeterminicity can be an alternative choice.**
Uncertainty of Knowledge/ Information	**Better than Incompleteness.**
Illogicalness of Logic Inconsistency of Form	**Non-rationality is a choice. Better than Incompleteness.**

Essential Uncertainties

Incompleteness of Form: approximate, complex, different, disharmonious, disorderly, diverse, equivocal, free, imperfect, improper, inapposite, incomplete, incongruent, inconsonant, inconstant, incorrect, indefinite, indistinguishable, irregular

69% with negative prefixes of 35 adjectives:

Vagueness of Consciousness: absent, anxious, blankly, blurred, changeable, confusing, dark, deceptive, dim, doubtful, dreamy, dubious, evasive, faint, hazy, imaginative, incredulous, indescriable, inexplicable, obscure, vague, wooly, woozy

6% with a negative prefix of 35 adjectives:

What do these mean?

'Consciousness' is uncertain in its natural state, while 'form' is certain in its natural state.

Three Essential Uncertainties and Their Mathematical Theories

(1) Probability of Phenomenon

Probability theory: Pascal and Fermat (17 C).

(2) Fuzziness of Wording

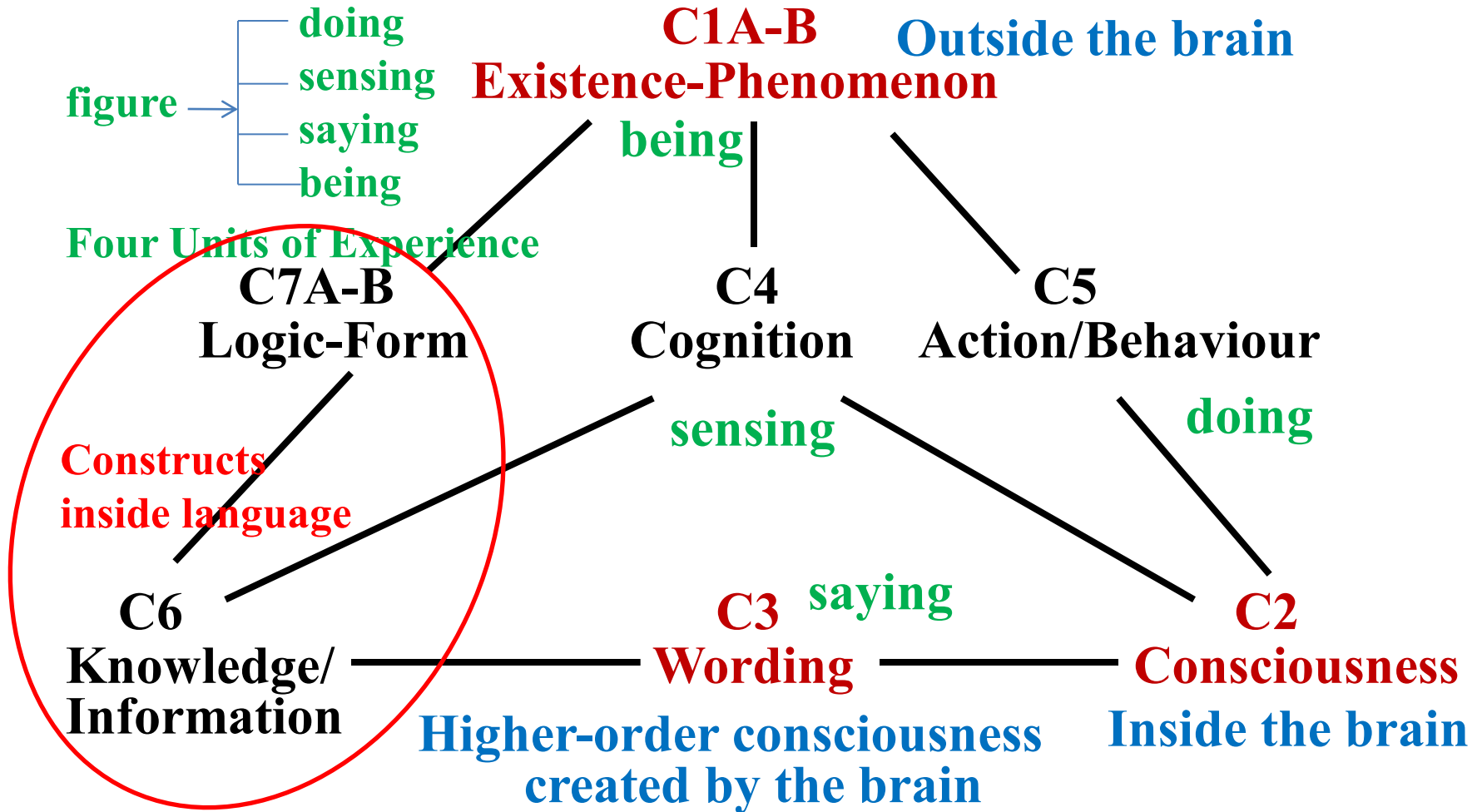**Fuzzy set theory: Zadeh (1965).**

(3) Vagueness of Consciousness

**The ultimate uncertainty for our species ?
No mathematical theory ?**

Theorization of human consciousness would be impossible.

Structure of Uncertainty (Sugeno 1986, 2011)

Verification of the Structure

1. Conducted **psychological experiments** with five student-subjects to obtain similarities of adjectives by pair-wise comparisons.

2. Applied **'Multi-Dimensional Scaling for Qualitative Data'** to the experimentally obtained data and allocated adjectives on 2D space concerning the first and second eigenvalues.

3. Then applied **the cluster analysis with shortest distance method** to the above 2D data to identify the structure of uncertainty.

MDS: Draw a map on 2D space based on city-distances

	Arad	Szeged	Oradea	Debrecen
Arad	0	105	115	190
Szeged	x	0	212	94,2
Oradea	x	x	0	75
Debrecen	x	x	x	0
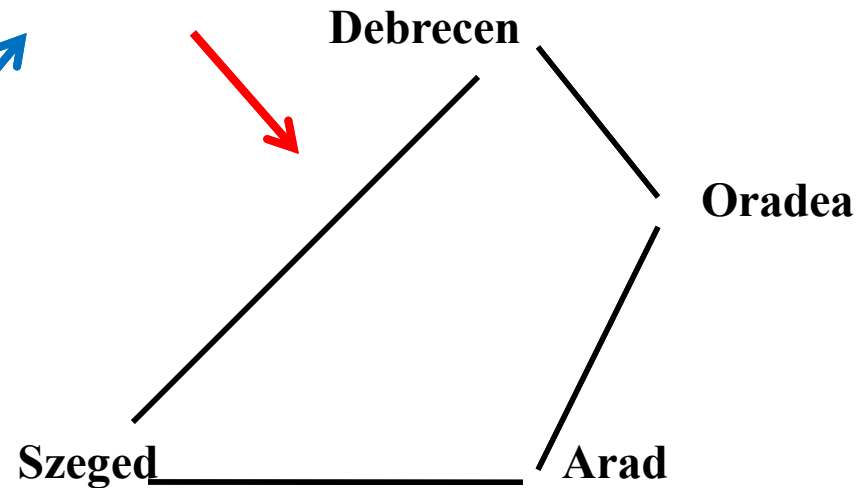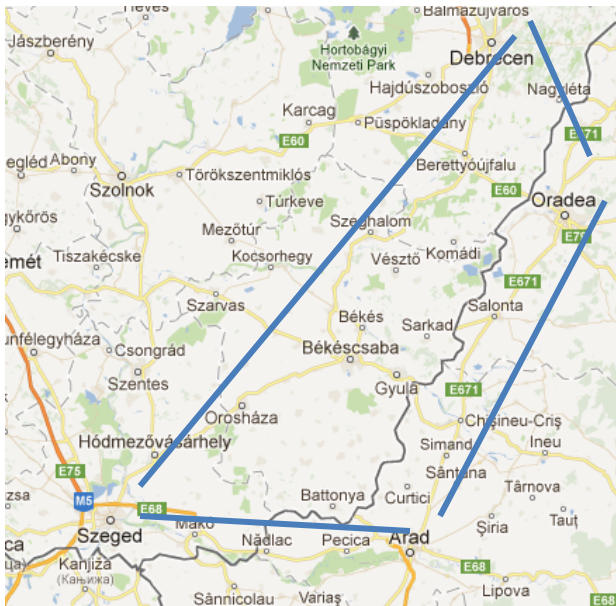
Identified Structure of Uncertainty
(Takayanagi, Yamanoi & Sugeno 1992)

Existence

Phenomenon

Logic

Form

Action/Behaviour

Cognition

Knowledge/
Information

Wording

Consciousness
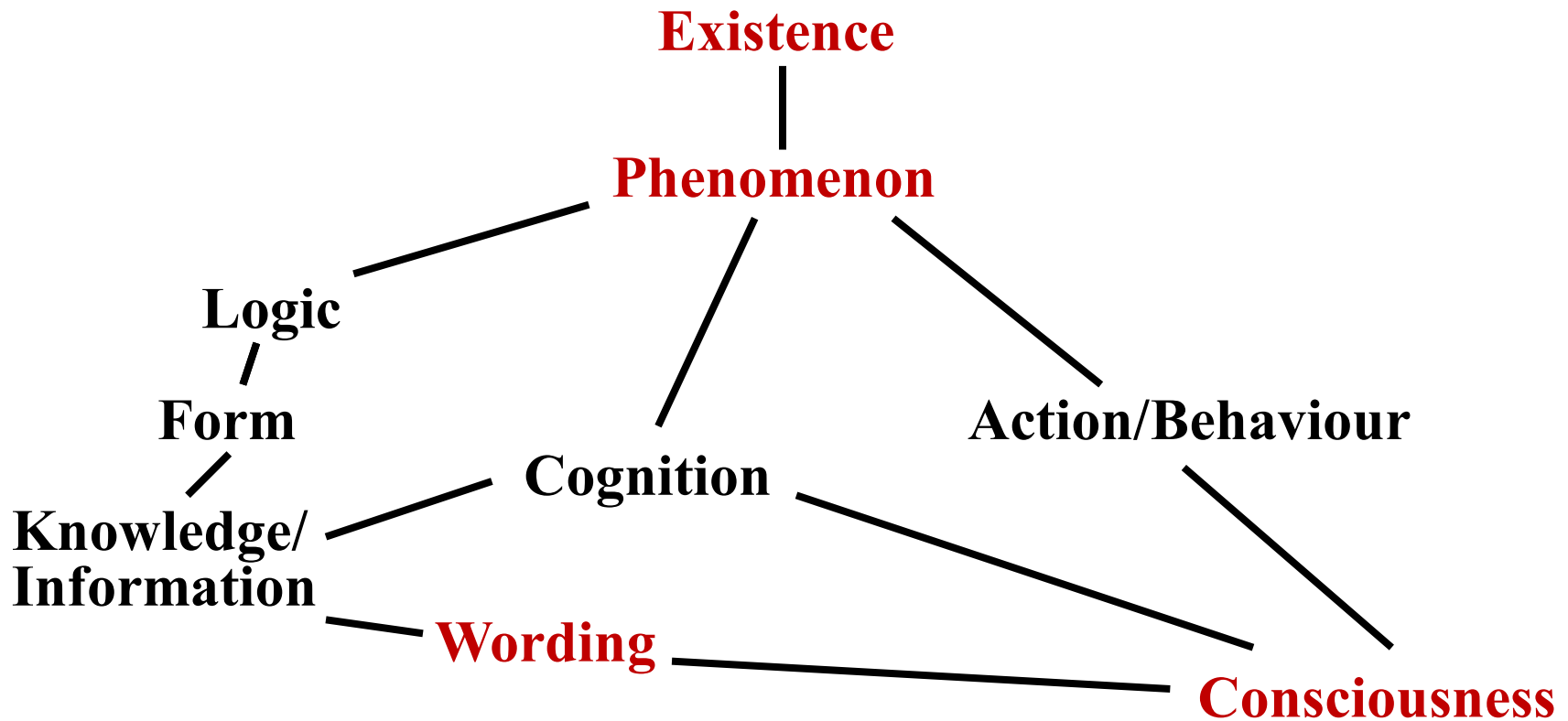
Their mutual distances were also identified.

Verification of Dominant Adjectives (1)
－A large-scale corpus analysis－

uncertain, obscure and vague

BNC: British National Corpus

    100 million written and spoken words (the number of English words
= about 1 million)

BCCWC: Balanced Corpus of Contemporary Written Japanese

    100 million written words (the number of Japanese words
= about a quarter million)

Corpus analysis in Quantitative Linguistics:

  Frequency Analysis: counting the number of occurrences of a word,
a phrase, etc.

  Co-occurrence Analysis: finding examples of co-occurrence of a specific
word with another word

  Concordance Analysis: finding context in use of a word, ex., KWIC (Key
Word In Context)

Frequency Analysis of Adjectives by BNC

1. various	15291	11. insufficient	1329	aleatoric	2
2. complex	9384	12. misleading	1303	disharmonious	2
3. unknown	5706	13. selective	1298	inconsequent	2
4. unlikely	5543	14. mysterious	1286	inconsecutible	1
5. anxious	2949	15. inappropriate	1230	inexhaustive	1
6. multiple	2446	16. probable	1182	multivocal	1
7. unexpected	2022	17. obscure	1175	inapprehesible	0
8. uncertain	1951	18. halfway	1066	incognizable	0
9. imaginative	1502	19. optional	959	incomprehensive	0
10. vague	1446	20. unclear	949	inequable	0

Zipf's law: $p(r) = \text{const.} / r$: (r = rank, $p(r)$ = frequency)

Thesaurus Analysis: Synonyms and Related Words

[Vague]   **5 (synonyms) + 39 (related words) = 42**

Synonyms: fuzzy, indefinite, inexplicit, muzzy, unclear.

Related Words: ambiguous, cryptic, dark, enigmatic equivocal, murky, nebulous, obscure, …. uncertain, undefinable, undefined, undetermined, …

[Uncertain]   **18 + 23 = 41**

Synonyms: capricious, changeable, changeful, flickery, fluctuating, fluid, inconsistent, inconstant, mercurial, mutable, skittish, temperamental, …

Related Words: aimless, arbitrary, desultory, erratic, haphazard, irregular, random, scattered, slapdash, stray, ambivalent, hesitating, shaky, vacillating, …

[Obscure]   **18 + 68 = 86**

Synonyms: ambiguous, arcane, cryptic, dark, deep, Delphic, elliptical, enigmatic, equivocal, fuliginous, inscrutable, murky, mysterious, mystic, …

Related Words: abstruse, esoteric, recondite, cloaked, concealed, disguised, masked, … uncertain, undefined, undetermined, vague, impenetrable, incomprehensible, indecipherable, inexplicable, eerie, uncanny, weird, impalpable, …

Translation into Other Languages

	Ara.	Chi.	Ger.	Fre.	Hun.	Ind.	Jap.	Hin.	Tur.	Vie.	AV
vague	4	12	14	6	2	10	24	2	6	5	7.5
uncertain	7	10	15	2	1	8	14	6	8	5	7.6
obscure	9	13	12	7	5	7	7	3	10	9	8.2
various	7	5	6	4	2	11	13	4	4	2	5.8
complex	3	9	6	1	2	4	3	2	5	4	3.9
unknown	2	7	2	1	1	1	6	8	6	1	3.5
dreamy	3	2	5	4	2	7	3	4	9	4	3.7
chaotic	3	1	3	1	2	8	1	7	3	2	3.1
ambiguous	3	4	3	2	2	6	6	2	4	3	3.5

Ara.: Arabic; Chi.: Chinese; Fre.: French; Hun.: Hungarian; Jap.: Japanese; Hin.: Hindi; Ind.: indonesian; Tur.: Turkish; Vie.: Vietnamese; and AV: Average .

The ten languages cover almost all <u>language families</u>.

Language Families in the World

- **Indo-European: Germanic (German**, Dutch), **Balto-Slavic** (Polish, **Russian), Italic (French, Rumanian)**, Celtic (Irish), Hellenic (Greek), **Indo-Iranian (Sanskrit, Iranian)**

- **Semitic** (Hebrew, **Arabic); Ham (Egyptian);**

- **Sino-Tibetan (Chinese,** Thai) ; **Uralic (Hungarian);**

- **Altaic (Turkish,** Mongolian, **Japanese);**

- **Malayo-Polynesian (Indonesian,** Hawaiian);

- **Austro-Asian (Vietnamese);** African; American-Indian; **Australian;** Dravidian; Caucasian;

- Paleo-Asiatic (Ainu itak); Eskimo-Aleut;

- **Basque (the most isolated in languages of the world)**

World Language Map

Language requirements for technical documentation

- English
- German
- Spanish
- French
- Portuguese
- Russian
- Japanese
- Chinese (Simplified)
- Chinese (Traditional)
- Arabic
- Other languages

Verification of Dominant Adjectives (2)
(Sugeno 2011)

Adjectives	Translation into * Other Languages	Frequency Analysis **	Synonyms/ *** Related Words	Domi-nancy
vague	7.5	1446	5 / 37 = 42	○
uncertain	7.6	1951	18 / 25 = 43	○
obscure	8.2	1176	17 / 37 = 54	◎
various	5.8	15291	17 / 40 = 57	◎
complex	3.9	9384	18 / 20 = 43	△
unknown	3.5	5706	5 / 12 = 17	×
dreamy	3.7	141	27 / 42 = 69	×
chaotic	3.1	460	20 / 49 = 69	×
ambiguous	3.5	837	17 / 67 = 84	×

*Google translation, ** BNC , *** Webster

RED: top ranking, BLUE: second ranking and BLACK: other rankings

Conclusions

(1) We have collected adjectives implying various uncertainties in seven languages: English, Japanese, Chinese, German, French, Spanish, and Basque.

(2) The number of adjectives differs depending on a language; 430 is counted in the case of English.

(3) Classifying these adjectives by the KJ method, we have found seven categories of uncertainty, or nine (if sub-categories are counted), which are associated with modalities of uncertainty.

(4) Allocating the categories on 2D space, we have found structure of uncertainty.

(5) The results have been verified by experiments and corpus analysis.

(6) We have also discussed some linguistic and logico-philosophical issues.

Future Studies

(1) Conduct **Psychological Experiments** with a variety of subjects (based on statistics) in representative languages selected from among language families to measure similarities of adjectives.

(2) Define dominancy among adjectives implying uncertainty and identify a couple of dominant adjectives.

(3) Identify nouns belonging to each category and find adjectives associated with each noun by applying **Co-occurrence Analysis** and **Concordance Analysis** to a large-scale corpus.

(4) Also find dominant adjectives in each category.

(5) Identify structure of uncertainty in each language and compare them to elucidate **common structure.**